Area Spectrum of Near Extremal Black Branes from Quasi-normal Modes

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Abstract

Motivated by the recent interest in quantization of black hole area spectrum, we consider the area spectrum of near extremal black $3-$branes. Based on the proposal by Bekenstein and others that the black hole area spectrum is discrete and equally spaced, we implement Kunstatter's method to derive the area spectrum for the near extremal black $3-$branes. The result for the area of event horizon although discrete, is not equally spaced.

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1 Introduction

Dynamical properties of a thermal gauge theory are encoded in its Green's functions. In the context of AdS/CFT [1], Minkowski space Green's functions can be computed from gravity using the recipe given in [2]. Unfortunately, for a non-extremal background, only approximate expressions for the correlators are usually obtained. Even thought the retarded Green's function in $4d$ cannot be found explicitly, the location of its singularities can be determined precisely. As shown in [2], this amounts to finding the quasinormal frequencies of dilaton's fluctuation in the dual near-extremal black brane background as functions of the spatial momentum. The possibility of a connection between the quasinormal frequencies of black holes and the quantum properties of the entropy spectrum was first observed by Bekenstein [4], and further developed by Hod [5]. Bekenstein noted that Bohr's correspondence principle implies that frequencies characterizing transitions between energy levels of a quantum black hole at large quantum numbers correspond to the black hole's classical oscillation (quasinormal) frequencies (see also [6, 7]). In particular, Hod proposed that the real part of the quasinormal frequencies, in the infinite damping limit (i.e. the $n \to \infty$ limit), might be related via Bohr's correspondence principle to the fundamental quanta of mass and angular momentum (see also [8]–[18]).
In asymptotically flat spacetimes the idea of QNMs started with the work of Regge and Wheeler [19] where the stability of a Schwarzschild black hole was tested, and were first numerically computed by Chandrasekhar and Detweiler several years later [20]. Recently, there has been considerable interest in studing the quasinormal modes in different contexts: in AdS/CFT duality conjecture [21]–[29], when considering thermodynamic properties of black holes in loop quantum gravity [30]–[32], in the context of possible connection with critical collapse [21, 25, 33], also when considering the area spectrum of black holes [34]-[39]. Recently it has been observed that the quasinormal modes can play a fundamental role in Loop Quantum Gravity [30]. Dreyer showed that in order to have consistence between the Bekenstein-Hawking entropy calculation and QNM frequencies, one had to assume that the minimum of j of the spin network piercing the horizon and contributing significantly to the entropy had to be $j = 1$. Whit this choice, the resulting Immrizi parameter would be given by $\gamma = \frac{\ln 3}{2\pi\pi\sqrt{2}}$. He suggested that if the gauge group of the theory were changed from $SU(2)$ to $SO(3)$ then this requirement would be immediately satisfied.
For the Schwarzschild black hole in four dimensions, the asymptotic real part of the gravitational quasinormal frequencies is of the form $\omega = T_H ln3$ where T_H is the Hawking temperature [40]. The suggestion of Hod was to identify $\hbar\omega$ with the fundamental quantum of mass ΔM. This identification immediately leads to an area spacing of the form $\Delta A = 4\hbar ln3$. An elegant approach, for the schwarzschild black hole in $d-$dimensions, based on analytic continuation and computation of the monodromy of the perturbation was proposed in [41], (to see more recent works refer to [42]).
In the present paper we extend directly the Kunstatter's approach [31] to determine mass and area spectrum of the near extremal black $3-$branes . According to this approach, an adiabatic invariant $I = \int \frac{dE}{\omega(E)}$, where E is the energy of system and $\omega(E)$ is the vibrational frequency, has an equally spaced spectrum, i.e. $I \approx n\hbar$, applying the Bohr-Sommerfeld quantization at the large n limit.

2 Coincident D-3 Branes

We consider now the background (in the string frame) of a black hole describing a number of coinciding D-3 branes [43, 44]

$$ds^2 = H^{-1/2}(r)[-f(r)dt^2 + \sum_{i=1}^{3}(dx^i)^2] + H^{1/2}[f^{-1}(r)dr^2 + r^2 d\Omega_5^2], \tag{1}$$

where

$$H(r) = 1 + \frac{l^4}{r^4}, \qquad f(r) = 1 - \frac{r_0^4}{r^4}, \tag{2}$$

and $d\Omega_5^2$ is the metric of a unit 5−dimensional sphere. The horizon is located at $r = r_0$ and the extremality is achieved in the limit $r_0 \to 0$. For l much larger than the string scale $\sqrt{\alpha'}$, the entire 3−brane geometry has small curvatures every where and is appropriately described by the supergravity approximation to type IIB string theory [43]. The requirement $l \gg \sqrt{\alpha'}$ translates into the language of $U(N)$ SYM theory on N coincident $D-3$branes. To this end it is convenient to equate the ADM tension of the extremal 3−brane classical solution to N times the tension of a single $D3-$ brane. Then one can find [45]

$$\frac{2l^4\Omega_5}{k^2} = N\frac{\sqrt{\pi}}{k}, \tag{3}$$

where $\Omega_5 = \pi^3$ is the volume of a unit 5−sphere, and $k = \sqrt{8\pi G}$ is the 10−dimensional gravitational constant. Therefore

$$l^4 = \frac{kN}{2\pi^{5/2}}. \tag{4}$$

In the other hand we have

$$k = 8\pi^{7/2} g_s \alpha'^2, \tag{5}$$

where g_s is the string coupling, then we obtain

$$l^4 = 4\pi N g_s \alpha'^2. \tag{6}$$

The parameters l and r_0 are related to the ADM mass in the following way

$$M = \frac{\Omega_5 V_3}{2k_{10}^2}(5r_0^4 + 4l^4). \tag{7}$$

Now we would like to consider the near-extremal 3−brane geometry. In the near-horizon region, $r \ll l$, we may replace $H(r)$ by $\frac{l^4}{r^4}$. The resulting metric is as following

$$ds^2 = \frac{r^2}{l^2}[-(1 - \frac{r_0^4}{r^4})dt^2 + d\overrightarrow{x}^2] + \frac{l^2}{r^2}(1 - \frac{r_0^4}{r^4})^{-1}dr^2 + l^2 d\Omega_5^2. \tag{8}$$

The above metric is a product of S^5 with a certain limit of the Schwarzschild black hole in AdS_5. The 8−dimensional area of the horizon can be read off from metric (8). If the spatial volume of the $D3-$brane is taken to be V_3, then we find

$$A_h = (\frac{r_0}{l})^3 V_3 l^5 \Omega_5 = \pi^6 l^8 T^3 V_3, \tag{9}$$

where T is a temperature

$$T = \frac{r_0}{\pi l^2}. \tag{10}$$

Using (4) we arrive at the Bekenstein-Hawking entropy [45]

$$S_{BH} = \frac{2\pi A_h}{k^2} = \frac{\pi^2 N^2 V_3 T^3}{2}. \tag{11}$$

3 Quasinormal Modes and Area Spectrum

Given a system with energy E and vibrational frequency $\omega(E)$, one can show that the quantity

$$I = \int \frac{dE}{\omega(E)} \tag{12}$$

where $dE = dM$, is an adiabatic invariant [31] and as already mentioned in the Introduction, via Bohr-Sommerfeld quantization has an equally spaced spectrum in the large n limit

$$I \approx n\hbar \ . \tag{13}$$

The large n asymptotic behavior of quasinormal frequencies given by following expression[3]

$$\omega_n^{\pm} = \omega_0^{\pm} \pm 2\pi T n(1 \mp i), \tag{14}$$

where

$$\omega_0^{\pm} = \pi T \lambda_0^{\pm}, \tag{15}$$

where $\lambda_0^{\pm} \approx \pm 1.2139 - 0.7775i$. Now by taking ω_R as

$$\omega_R = \pm \pi T(2n + 1.2139), \tag{16}$$

then by substituting Eq.(10) we get

$$\omega_R = \frac{\pm r_0}{l^2}(2n + 1.2139). \tag{17}$$

By taking M as given by Eq.(7) and substituting Eq.(6), we obtain

$$M = \frac{V_3}{128\pi^4 \alpha'^4 g_s^2}(5r_0^4 + 16\pi N g_s \alpha'^2). \tag{18}$$

Then, the parameter r_0 is given by

$$r_0 = (aM - b)^{1/4}, \tag{19}$$

where

$$a = \frac{128\pi^4 \alpha'^4 g_s^2}{5V_3}, \qquad b = \frac{16\pi N \alpha'^2 g_s}{5}. \tag{20}$$

Now by taking ω_R as given by expression (17) and substituting Eqs.(19,6), we get

$$\omega_R = \frac{\pm(aM - b)^{1/4}}{2\alpha' \sqrt{\pi N g_s}}(2n + 1.2139). \tag{21}$$

Thus, the adiabatically invariant integral (12) is written as

$$I = \frac{\pm 2\alpha' \sqrt{\pi N g_s}}{(2n + 1.2139)} \int \frac{dM}{(aM - b)^{1/4}}, \tag{22}$$

and after integration, we obtain

$$I = \frac{5V_3\sqrt{N}}{48\pi^{7/2}\alpha'^3 g_s^{3/2}(2n + 1.2139)}(aM - b)^{3/4}. \tag{23}$$

Now using Eqs.(10,19) we can rewrite the area of the horizon Eq.(9) as

$$A_h = \pi^3 l^2 V_3 r_0^3 = \pi^3 l^2 V_3 (aM - b)^{3/4}. \tag{24}$$

Using Eq.(23) the area is given by following expression

$$A_h = \frac{3(2n + 1.2139)}{160}\pi^7 \alpha'^4 g_s^2 I = \frac{3}{160}\pi^7 \alpha'^4 g_s^2 (2n + 1.2139) n\hbar. \tag{25}$$

It is obvious that the area spectrum, although discrete, is not equivalently spaced. The quasinormal frequencies which given by Eq.(14) was later generalized in the paper by Nunez and Starinets [46], Eqs. (3.22-3.23), but all these formulas are asymptotics rather than exact results. However, for vector perturbations (and spatial momentum on the brane equal to zero) the spectrum turns out to be exact and given by following relation

$$\omega_n = n(1 - i), \qquad n = 0, 1, ... \tag{26}$$

therefore, the integral Eq.(12) yields

$$I = \frac{M}{n}, \tag{27}$$

and by equating expressions (27) and (13), we get

$$M = n^2 \hbar. \tag{28}$$

It is obvious that the mass spectrum of $3-$black brane is quantize.

4 Conclusion

The possibility of a connection between quasinormal modes and the area spectrum of black holes has been actively pursued over the past year. Many examples have been studied, and progress has been made towards a general understanding of this connection. Bekenestein's idea for quantizing a black hole is based on the fact that its horizon area, in the nonextremal case, behaves as a classical adiabatic invariant. It is interesting to investigate how near extremal black $3-$branes would be quantized. Discrete spectra arise in quantum mechanics in the presence of a periodicity in the classical system, which in turn leads to the existence of an adiabatic invariant or action variable. Boher-Somerfeld quantization implies that this adiabatic invariant has an equally spaced spectrum in the semi-classical limit. Kunstatter showed that this approach works for the Schwarzschild

black holes in any dimension, giving asymptotically equally spaced areas, previously we have showed generalization to non-rotating BTZ, extremal Reissner-Nordström, near extremal Schwarzschild-de Sitter Kerr and extremal Kerr black holes [34, 35, 36, 37] is also successful. In this article we have considered the near extremal black $3-$branes. Using the results for highly damped quasi-normal modes Eq.(14), we obtained the area spectrum of event horizon Eq.(25), which is obvious that the area spectrum, although discrete, is not equivalently spaced. Using the generalized form of quasinormal modes Eq.(26), we obtained mass spectrum of near extremal $3-$black brane as Eq.(28). Similar situation occur for BTZ black hole[34] which the area of event horizon is not equally spaced, in contrast with area spectrum of black hole in higher dimension, although the mass spectrum is equally spaced.

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